QUANTITATIVE METHODS

TIME VALUE OF MONEY

Interest rate = Risk free rate + Inflation risk premium + Default risk premium + Liquidity risk premium + Maturity risk premium

FV_t = Future Value (period t) = \$X x $(1+r)^t$

PV_0 = Present Value (time 0) = $FV_t / (1+r)^t$

Stated Annual Rate = $SAR = r_p * n_p$;

r_p = Periodic rate, and

n_p = Number of periods in a year

Effective Annual Rate = $EAR = (1 + r_p)^{n_p} - 1$

FVA_T = Future Value of an Annuity =

$$= \frac{PMT\ x[(1+r)^T - 1]}{r}$$

PVA_0 = Present Value of an Annuity

$$= \frac{PMT\ x\ [1 - \frac{1}{(1+r)^T}]}{r}$$

$FVAD_T$ = Future Value of Annuity Due

$$= PMT\ x \frac{[(1+r)^T - 1]}{r} = (1+r)$$

$PVAD_T$ = Present Value of Annuity Due

$$= PMT\ x \frac{[1 - \frac{1}{(1+r)^T}]}{r} = (1+r)$$

Present Value of Perpetuity = $\frac{PMT}{r}$

Future Value of Uneven Cash Flows =

$$FV_T = \sum_{t=0}^{T} C_t (1+r)^{T-t}$$

Present Value of Uneven Cash Flows =

$$PV_0 = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}$$

DISCOUNTED CASH FLOW APPLICATIONS

Net Present Value = $NPV = \sum_{t=0}^{T} \frac{CF_t}{(1+r)^t}$

For IRR, solve: $NPV = \sum_{t=0}^{T} \frac{CF_t}{(1+IRR)^t} = 0$

If **IRR** ≥ k, **Accept**; If **IRR** < k, **Reject**.
where **k** = Cost of Capital

Money Weighted Return is an IRR calculation.

Holding Period Return =

$$\textbf{HPR} = \frac{V_{END} - V_{BEG} + CF_{END}}{V_{BEG}}$$

TWR=$[(1+HPR_1)(1+HPR_2)..(1+HPR_n)]^{1/n}$-1
Then annualize the time-weighted return.

Bank Discount Yield = **BDY** =

$$\left[\frac{\text{Face Value - Market Price}}{\text{Face Value}}\right] x \left(\frac{360}{T}\right) x 100$$

Money Market Yield =

$$\textbf{MMY} = \frac{\text{Discount}}{\text{Price}} x \frac{360}{\text{T}} x 100$$

Bond Equivalent Yield =

$$\textbf{BEY} = \text{HPY}\ x\ \frac{360}{T}$$

STATISTICAL CONCEPTS AND MARKET RETURNS

Gross Return = $R = \frac{P_1 + CF_1}{P_0}$

Average gross return=$(R_1\ x\ R_2\ x\ ...\ x\ R_T)^{1/T}$

Arithmetic Mean = $\bar{Y} = \frac{1}{n} \sum_{i=1}^{N} Y_i$

Weighted Mean = $\bar{Y}_W = \sum_{i=1}^{N} W_i Y_i$, where W_i are weights that sum to 1.

Geometric Mean =

$$\bar{Y}_G = (Y_1\ x\ Y_2\ x\ Y_3\ x\ g\ g\ g\ x\ Y_N)^{1/N}$$

Harmonic Mean = Inverse of arithmetic average of inverse of observations.

z^{th} percentile = $P_z = (N+1)\frac{z}{100}$

Mean Absolute Deviation = MAD =

$$\frac{1}{N} \sum_{i=1}^{N} |Y_i - \bar{Y}|$$

Population Variance = $\frac{1}{N} \sum_{i=1}^{N} (Y_i - \bar{Y})^2$

Sample Variance = $\frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$

Standard Deviation = √(Variance)

Target semi-variance =

$$\sum_{n=1}^{T'} \frac{\left(X_n - X_{TGT}\right)^2}{T'-1}; \text{such that } X_n \le X_{TGT}$$

for *semi-variance*, replace $X_{TGT} = \bar{Y}$

Chebyshev's inequality: Proportion of observations in a range k ($k > 1$) standard deviations around the mean $\ge 1 - \frac{1}{k^2}$

Sharpe measure = $\frac{\bar{R}p - \bar{R}F}{\sigma_p}$

Sample Excess Kurtosis =

$\frac{1}{N}\frac{\sum_{i=1}^{N}(Y_i - \bar{Y})^4}{s^4} - 3$; Kurtosis of normal distribution = 3.

PROBABILITY CONCEPTS

Positive skewness: Mode < Median < Mean

Negative skewness: Mode > Median > Mean (Positive skewness is preferred)

Joint probability=$p(E_1E_2) = p(E_1 \mid E_2)\, p(E_2)$

Addition rule of probabilities =

$$p(E_1 \text{ or } E_2) = p(E_1) + p(E_2) - p(E_1E_2)$$

Multiplication rule for independence =

$$p(E_1E_2) = p(E_1 \mid E_2)\, p(E_2) = p(E_1)\, p(E_2)$$

Total Probability Rule = $p(\boldsymbol{C}) = p(C_1 \mid E_1)\, p(E_1) + p(C_2 \mid E_2)\, p(E_2) + \ldots + p(C_n \mid E_n)\, p(E_n)$

Bayes Formula = $p(E_1 \mid E_2) = \frac{p(E_2E_1)}{p(E_2)}$

Expected Value = $E(\tilde{Y}) = \sum_{s=1}^{n} p_i Y_i$

Variance (based on probability distribution)

$$= \sigma^2 = \sum_{i=1}^{n} p_i\left[Y_i - E(\tilde{Y})\right]^2 = E\left[Y_i - E(\tilde{Y})\right]^2$$

Conditional Expectation =

$$E(\tilde{Y}) = p(Y_1 \mid I)Y_1 + \text{ggg} + p(Y_n \mid I)Y_n = \sum_{i=1}^{n} p(Y_i \mid I)Y_i$$

Expected Value (Total Probability Rule) =

$$E(\tilde{Y}) = E(Y \mid I_1)\, p(I_1) + \text{ggg} + E(Y \mid I_K)\, p(I_K)$$

Variance (using conditional expectations) =

$$\sigma^2 = \sum_{i=1}^{n}\left[E(R \mid I_i) - E(R)\right]^2 p(I_i)$$

Variance = $\sigma^2 = \sum_{i=1}^{n} p_i\left[Y_i - E(\tilde{Y})\right]^2$

Covariance = $\sum_{i=1}^{n} p_i\left[Y_i - E(\tilde{Y})\right]\left[Z_i - E(\tilde{Z})\right]$

Correlation coefficient = $\rho_{YZ} = \frac{\text{Cov}_{YZ}}{s_Y s_Z}$

Portfolio Exp. Ret. = $E(R_p) = \sum_{s=1}^{n} w_s E(R_s)$

Portfolio Variance =

$$\sigma_p^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_Aw_BCov_{AB}$$

Multiplication Rule of Counting =

$$n_1 \text{ x } n_2 \text{ x } n_3 \text{ x } n_4 \text{ x } n_5 \text{ x } \ldots \text{ x } n_k$$

Multinomial Formula =

$$\frac{n!}{n_1!\, x\, n_2!\, x\, n_3!\, x\, n_4!\, x \ldots x\, n_k!}$$

Combinatorial Formula=${}^{n}C_r = \frac{n!}{r!\, x\, (n-r)!}$

Permutation Formula = $\frac{n!}{(n-r)!}$

Application Rule: If number of outcomes is *infinite*, *cannot use* any counting method. If n objects: n slots, all objects assigned to one slot, use *factorial* formula. If assigned to *three* or more groups, use *multinomial* formula. If r objects selected *without* order, use *combinatorial* formula. If r objects selected *with* regard to order, use *permutation* formula. Otherwise investigate use of *multiplication* rule, or actual count.

PROBABILITY DISTRIBUTIONS

Binomial Probability = p (r successes) =

$$\frac{n!}{r!\, x\, (n-r)!}\ x\ (p)^r\, x\, (1\text{-}p)^{n-r}$$

***E*(Success)** = $\mu = np$

$\sigma^2 = np(1\text{-}p)$

Continuous Uniform Distribution: *pdf* =

$$f(y) = \begin{cases} \frac{1}{y_U - y_L} & \text{for } y_L \le y \le y_U \\ 0 & \text{otherwise} \end{cases}$$

Properties of Normal Distribution:

1. Completely described by mean and variance (μ, σ^2)
2. It is symmetric with skewness measure of 0, i.e., *mean* = *mode* = *median*
3. Kurtosis = 3. Forms benchmark.
4. Linear combinations of normal random variables are normally distributed.

Confidence Interval for Sample Mean:

$$[\bar{Y} - z\, s/\sqrt{n},\ \bar{Y} + z\, s/\sqrt{n}\ ]$$

Standard Normal Variate:

$$z = \frac{y - \bar{Y}}{s}$$

Safety First Ratio $= \dfrac{E(R_p) - R_T}{\sigma_p}$

Lognormal Distribution:

$$\ln(\bar{Y}_{\%}) \sim N(\mu, \sigma^2)$$

Continuously compounded rate $= e^{rt} - 1$

SAMPLING & ESTIMATION

Central Limit Theorem: Irrespective of the underlying distribution, sample means ($\bar{Y}$) based on sample size (n), have the same mean (μ) as the underlying population, and variance that equals σ^2/n, where σ^2 is the variance of the underlying population. The sample means are approximately normally distributed large sample sizes (≥30).

Standard Error of Sample Mean:
$\sigma_{\bar{Y}} = \sigma/\sqrt{n}$ (population variance known)
$s_{\bar{Y}} = s/\sqrt{n}$ (population variance unknown)

Estimator Properties: Unbiasedness, Efficiency & Consistency.

Student's *t*-distribution: is symmetric; fatter tails; and, is characterized by degrees of freedom.

Confidence Interval for Mean:

$$\bar{X} \pm t_{\alpha/2,\ df}\ (s/\sqrt{n})$$

HYPOTHESIS TESTING

Null Hypothesis	Test type
Equality	Two-tailed
Inequality	One-tailed

Test Statistic (test of mean, normal distbn.)

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

Type I Error: Null is Rejected when True

Type II Error: Null is Accepted when False

Power of Test = 1 – Type II Error

Test of Mean:

$$z = \frac{\bar{Y} - \mu_0}{s/\sqrt{n}};\ t_{n-1} = \frac{\bar{Y} - \mu_0}{s/\sqrt{n}},\ \text{(n-1) } d.f.$$

Test of Equality of Means:

1. Unknown population variance assumed equal

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 + n_2 - 2)}$$

$$t_{n_1+n_2-2} = \frac{(\bar{Y}1 - \bar{Y}2) - (\mu_1 - \mu_2)}{s\left(\frac{1}{n_1} + \frac{1}{n_2}\right)^{1/2}}$$

2. Unknown variances assumed unequal

$$t_{df} = \frac{(\bar{Y}1 - \bar{Y}2) - (\mu_1 - \mu_2)}{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^{1/2}}$$

$$modified\ df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(s_1^2/n_1\right)^2}{n_1} + \frac{\left(s_2^2/n_2\right)^2}{n_2}}$$

Matched Pair Test:

$$d_i = Y_{ai} - Y_{bi};\ \bar{d} = \sum_{i=1}^{n} d_i$$

$$s_d^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}(d_i - \bar{d})^2;\ s_{\bar{d}}^2 = \frac{s_d^2}{n}$$

$$t_{n-1} = \frac{\bar{d} - 0}{s_d/\sqrt{n}}$$

Test of Variance Against Point Value:

$$\chi^2 = \frac{(n\text{-}1)s^2}{\sigma_0^2};\ \text{(n-1) } d.f.$$

Test of Equality of Variances:

$$F_{n_L-1,n_S-1} = \frac{s_L^2}{s_S^2};\ d.f.\ (n_L\text{-}1)\ \&\ (n_S\text{-}1)$$

CORRELATION & REGRESSION

Test of Correlation (*r*) Against Zero:

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}};\ \text{(n-2) } d.f.$$

Linear Regression:

$$Y_i = c_0 + b_1 X_i + \varepsilon_i$$

Assumptions: Linearity, non-randomness, zero error mean, homoskedasticity, uncorrelated errors, normal errors.

Potential Problems: Heteroskedasticity, Multicollinearity, Autocorrelation.

Standard Error of Estimate:

$$SEE = \left[\frac{1}{(n-2)} \sum_{i=1}^{n} \left(\hat{\varepsilon}_i^{\,2} \right) \right]^{1/2}$$

Test of Significance of Co-efficient:

$$t = \frac{\hat{b}_1 - b_1^{\,0}}{s_{\hat{b}_1}} \text{; (n-k+1) } d.f.$$

Analysis of Variance:

TSS = RSS + SSE
MSE = SSE ÷ (n-k+1)
MSR = RSS ÷ k
R-sq = RSS ÷ TSS
r = √R-sq
F = (RSS ÷ k) / [SSE ÷ (n-k+1)]; k & (n-k+1) *d.f.* {Test of joint significance of coefficients, i.e., regression.}
SEE = √MSE

ECONOMICS

GDP AND NATIONAL INCOME

GDP (Expenditure Approach):

GDP = C + I + G + NX

GDP (Resource Cost–Income Approach):

GDP = R + T + D + NF

GNP = GDP - NF

Real GDP$_{\text{Year T}}$ =

$$Nominal\ GDP_T\ x\ \frac{GDP\ Deflator_{Base\ Year}}{GDP\ Deflator_T}$$

- *National Income* = *GNP* – Depreciation – Indirect taxes
- *Personal income* = National income – Corporate profits & social insurance taxes + Social security, net interest and dividends received
- *Disposable income* = Personal income – Personal taxes

Long Run Aggregate Supply: Constant in short run. Changes when economic resources or productivity change.

Real GDP = *Planned* consumption + *Planned* investment + *Planned* government expenditures + *Planned* net exports

Inflation rate = $\frac{Price\ Index_t - Price\ Index_{t-1}}{Price\ Index_{t-1}}$

Natural Rate of Unemployment = 1 – Full employment rate

Unemployment: Frictional, Structural & Cyclical

Fiscal Policy: (1) During a recession moves the AD curve to the right at full employment level output but at higher prices. (2) During boom times, AD is moved left for full employment output at lower prices.

Crowding Out Model: Fiscal deficits when financed by borrowing, raise interest rates. Neo-classical Model: When fiscal deficit is high to stimulate economy, people cut back spending in expecting higher future taxes.

Supplyside Model: Lowering tax rates increases productive resources and better tax collection, shifting LRAS to the right.

MONETARY POLICY

M1: Currency, checkable deposits and travelers' checks.

M2: M1 + savings deposits, money market deposits and small deposits under $100,000

Potential deposit expansion multiplier = *1/RRR*

Fed Policy Tools: Discount Rate; Reserve Ratio; Open Market Operation (most popular); Fed Funds Rate (tracking only)

Effect of Monetary Policy: Indirect through interest rates. Expansionary money supply in a recession lowers real interest rates spurring investment and consumption demand for durables, moving AD curve to the right.

Money Supply Change Anticipated: If its effect is perceived to be inflationary – higher price equilibrium results at same output.

Equation of Exchange: Quantity theory of money.

P *x* Q = M *x* V

Growth rate of real output + Rate of inflation = Growth rate of money supply + Growth rate of velocity

STABILIZATION & OUTPUT

Policy Time Lags: Recognition, Administrative, Impact.

Adaptive expectations: Adjustment process is slow and errors are systematic.

$$E(X_t) = \alpha X_{t-1} + (1-\alpha) X_{t-2}$$

Rational expectations: Assumes market considers all available information like policy makers reaching the same conclusion.
Activist Policy: Fiscal policy & monetary policy changes to smoothen business cycle.
Non-activist Policy: Stable inflation rate, stable tax rates and stable government expenditures, irrespective of the short-run state of the economy. Government's budget balanced over a business cycle. Steady monetary growth to accommodate growth in real output.

SUPPLY & DEMAND

Consumer Surplus: Area under the demand curve but above the equilibrium price.

Elasticity of Demand = % Change in Quantity ÷ % Change in Price (midpoints)

Producer Surplus: Area above the supply curve but below the equilibrium price.

Total surplus = Consumer surplus + Producer surplus
Elasticity of Supply = % Change in Quantity ÷ % Change in Price (midpoints)
Shifts Along Supply and Demand Curve: These are caused by changes in price.

Shifts in Demand Curve: Caused by factors other than price: consumer income, number of consumers, price of related good, changes in expectations, demographics, consumer tastes and preferences.
Shifts in Supply Curve: Caused by factors other than price: price of resources, technology, natural and political disruptions, tax changes, number of firms in the market.

Long Run Supply Curve: Is more elastic than the short run supply curve.

Shortages and Prices: In the short run price increases but in the long run both the demand curve and supply become more elastic and price falls somewhat.

Prices as Invisible Hand: Communicates information; Coordinates actions; Motivates market participants.
Price Ceilings & Floors: Ceilings create excess demand and floors create excess supply and reliance upon non-price factors.
Tax incidence: Greater proportion of actual incidence of tax is borne by the party with the more inelastic curve: demand or supply.

Choice Among Several Goods: Consumer equate:

$$\frac{MU_1}{P_1} = \frac{MU_2}{P_2} = \frac{MU_3}{P_3} = \ldots = \frac{MU_k}{P_k}$$

Determinants of Elasticity of Demand: Availability of substitutes; Impact of time; Share of budget spent on a product; Consumer's opportunity cost of time.
Change in Total Consumer expenditure:

$$\propto (1 + e_P)$$

Assumptions Behind Indifference Curves: (1) Prefer more over less, (2) Goods can be substituted, and (3) Marginal utility declines with consumption. No two indifference curves for an individual can intersect.
Substitution Effect: Price increase will lead to lower consumption of the good and switching by consumers to its substitutes.
Income Effect: Change in price of a good in has the same affect as a change in her income. Price increase will lead to a cutback in the consumption of the good.

COSTS AND SUPPLY

Economic Profit: It takes into account implicit and explicit costs.
Accounting Profit: Only takes into account explicit costs.
Marginal Product and Cost: Marginal product first increases with volume and then decreases. Marginal cost first decreases and then increases.

LRATC: This is the envelope of the all possible minimum cost points of short run average total cost.
Economies of Scale: As volume increases, average total cost declines.
Cost Curves Shift: Due to – Resource prices, Taxes, technology, Regulation.

PRICE TAKERS

Demand Curve: Industry demand curve is downward sloping and firm's demand curve is horizontal. Many firms, none can influence price.

Equilibrium Condition: $p_M = MR = MC$
If $p_M < AVC$, a firm would shut down temporarily.

Long Run Equilibrium: All firms make zero economic profits, and

$$p_M = MR = MC = ATC$$

PRICE SEARCHERS – LOW BARRIERS

Profit Maximizing Condition: Firms face a downward sloping demand curve and maximize profit by setting

MR = MC

Also, Marginal Revenue (MR) < Price

Contestable Market Implications: Due to threat of entry, firms make zero economic profit, and price = ATC.

Price Discrimination: Firms charge different prices in markets with different demand elasticities to increase revenue and profits.

PRICE SEARCHERS – HIGH BARRIERS

Barriers to Entry: (1) Economies of scale, (2) Government licensing, (3) Patent rights, (4) Control of a key resource by a firm.

Monopoly: Only one firm.

Oligopoly: More than one, usually less than five, firms.

Profit Maximizing Condition: MR = MC under both – oligopoly and monopoly.

Collusion Under Oligopoly: Price and profits are higher. Quantity is lower. Firm has incentive to cheat by increasing output. LRATC is flat and serves as MC curve.

Obstacles to Collusion: (1) Effectiveness decreases as firms increase, (2) Monitoring and detection difficulty, (3) Threat of entry if low barriers, (4) Unstable demand, and (5) Antitrust and legal prosecution.

Regulatory Problems in Controlling a Monopoly: (1) Imperfect information, (2) Shifting of costs, (3) Influence of special interests, and (4) Delayed response.

Natural Monopoly: It results when there are increasing returns to scale.

PRODUCTIVE RESOURCES

Demand for a Resource: Derived from demand for final products in which it is used. Assumes the characteristics of the final product's demand curve.

Effect on Elasticity: Two factors – substitution in production and substitution in consumption. Long run demand curve is more elastic.

Shift in Demand Curve: (1) demand for final product, (2) Productivity of resource, and (3) Price of related products.

Marginal revenue product (*MRP*) =

Marginal Revenue (*MR*) *x* Marginal Product (*MP*)

Profit Maximizing Condition:

MRP = *Price* (of additional resource unit), for *all resource types*

Profit maximizing or Cost Minimizing Condition:

$$\frac{\text{Marginal Product}_A}{\text{Price}_A} = \frac{\text{Marginal Product}_B}{\text{Price}_B} = \ldots$$

FINANCIAL ENVIRONMENT

Factors Affecting Supply & Demand for Capital: (1) Production opportunities, (2) Time preference for consumption, (3) Risk of investments, and (4) Inflation.

Nominal Risk Free Rate = Real risk free rate + Expected inflation rate

Risk Premiums for Bonds: (1) Default risk premium – due to possibility of default on promised payments, (2) Liquidity premium – due to lack of market for a security where it can be sold quickly at a fair price with low transactions cost, (3) Maturity risk premium – due to greater volatility of prices of long term bonds when interest rates change.

Interest Rate Structure:

$$k_{risky} = k_{real} + IP + DRP + LP + MRP$$

(IP – inflation premium)

Interest Rate Risk: Affect on bond prices when rates rise.

Reinvestment Rate Risk: Coupon and principal repayments will be reinvested at lower rates if rates fall.

GLOBAL ECONOMIC ANALYSIS

Law of Comparative Advantage: Among a group of countries, a country with the lowest opportunity cost of producing a good should be the one producing it.

Gains from International Trade: Impacts consumer and producer surplus. Imports increase consumer surplus at the expense of producers if world prices are lower. Exports increase producer surplus at the expense of consumers if world prices are higher.

Tariffs and Quotas: Tariffs increase producer surplus and government revenue at the expense of consumers. Quotas increase producer surplus and foreign exporters' revenues and create a vested interest.

FOREIGN EXCHANGE

Direct and Indirect Quotes: In a direct quote, foreign currency is quoted on terms of domestic currency. Indirect quote is the exact opposite. Also,

$$\text{Indirect quote} = (\text{Direct quote})^{-1}$$

Inverting Bid-Ask Quotes:

Indirect bid (fc/dc) = 1 / Direct ask (dc/fc), and vice versa

Indirect ask (fc/dc) = 1 / Direct bid (dc/fc), and vice versa

Bid-Ask Spread (percentage) = [(ask – bid) / ask] *x* 100. (Average of bid and ask may also be used in the denominator.)

Factors Affecting Spread: (1) Market conditions – volatility, (2) Trading volume, and (3) Dealers' inventory or position (only affects the midpoint, not the spread.)

Cross Rates:

$(fc_1/fc_2)_{ask} = (fc_1/dc)_{ask} \; x \; (dc/fc_2)_{ask}$, and

$(fc_1/fc_2)_{bid} = (fc_1/dc)_{bid} \; x \; (dc/fc_2)_{bid}$

Forward Percentage Spread = (Forward Ask – Forward Bid) ÷ [Forward Ask + Forward Bid)/2)]

Factors Affecting Forward Spread: (1) Market conditions – volatility, (2) Trading volume, (3) Dealers' inventory or position (only affects the midpoint, not the spread.), and (4) Contract maturity.

Forward Discount and Premium: In direct terms, if Spot (S) > F (Forward) rate, foreign currency is selling at a discount. Otherwise, it is selling at a forward premium.

Annualized Discount or Premium:

$$\text{D (or P)} = \left(\frac{S-F}{S}\right) x \frac{360}{T} x 100$$

Interest Rate Parity: Interest rates across countries are equal after adjusting them for their currency forward premium or discount.

Covered Interest Arbitrage: The following condition holds in equilibrium –

$$\frac{S}{F}(1+r_F) = (1+r_D).$$

If it is violated, arbitrage exists. Depending on which side less, buy that side and short the other side of the equation for an arbitrage profit. This brings market into equilibrium.

FOREIGN EXCHANGE PARITY

Balance Equation:

Current account balance + Financial account balance + Change in official reserves = 0

Growth and Account Balances: A growing economy may have a trade account deficit and a financial account surplus as it attracts foreign investment. If an economy is not growing, a large current account deficit results in a fall in its currency value due to excess supply of domestic currency.

Factors Affecting Currency Value: (1) Inflation rate, (2) Real rate, (3) Economic performance of a country, and (4) Investment climate of a country.

Effect of Monetary Policies on Currency:

Restrictive ⇒ Appreciation
Expansionary ⇒ Depreciation

Effect of Fiscal Policy on Currency:

Restrictive ⇒ Depreciation
Expansionary ⇒ Appreciation

(Interest rate effect dominates)

Relative Purchasing Power Parity:

$$S_F = S_0 \; x \; \frac{(1+I_{FC})}{(1+I_{DC})}; \text{ (Indirect quotes)}$$

Exchange Rate Systems: (1) Pegged system, (2) Managed system, and (3) Floating rate system.

FINANCIAL STATEMENTS

SHORT TERM LIQUID ASSETS

Securities:

- *Held to Maturity*: Not marked to market.
- *Trading Securities*: Marked to market and unrealized gains & losses taken to income statement.
- *Available for sal*e: Marked to market but unrealized gains & losses taken directly to shareholders/ equity

INVENTORY

COGS = Opening Inventory + Purchases – Closing Inventory

Inventory Costing Methods: (1) LIFO, (2) FIFO, (3) Weighted average, (4) Specific physical identification.

Balance Sheet & Income Effects: FIFO better for balance sheet and LIFO better for income statement.

LIFO reserve = $Inv_{FIFO} - Inv_{LIFO}$
LIFO Effect = Change in LIFO reserve.

Working Capital Effect = $Working\ capital_{FIFO - LIFO} = -T \times$ COGS difference + LIFO Reserve

LONG LIVED ASSETS

Depreciation Methods: (1) Straight line, (2) Units of production, (3) Accelerated – DDB and SYD (ignore salvage value), and (4) Annuity or sinking fund. Land is not depreciated.

Average depreciable life = Ending Investment (Gross) ÷ Depreciation expense

Average age = Accumulated depreciation ÷ Depreciation expense

Average age = Relative age x Average depreciable life

Exchange of Asset: (1) Gain and loss recognized for tax and GAAP for dissimilar asset, (2) Only loss recognized under GAAP for similar asset.

Oil & Gas Exploration: Costs capitalized or written off based on (1) Successful efforts, or (2) Full costing.

Intangibles: (1) Goodwill never recognized unless purchased and written down when impaired, (2) R&D costs expensed under GAAP, (3) Post-feasibility software costs can be capitalized and depreciated under straight line basis.

LONG TERM LIABILITIES

Premium and Discount Bonds: Premium over face or discount from face value is recognized as unamortized premium or discount and amortized periodically using straight line or effective interest method.

Deferred Tax Liability: When reported tax expense exceeds taxes payable.

Deferred Tax Asset: When taxes payable exceed reported tax expense.

Deferred Taxes: These may or may not be self reverting depending (1) future tax law changes, (2) accounting method changes, (3) growth rate of firm's assets, and (4) non-recurring items and adjustments to equity.

Capital Lease: If any one of following met:

- Bargain purchase option at end
- Ownership transferred to lessee at end
- Term at least 75% of economic life
- Present value of lease payments at least 90% of asset cost to lessor

Off-balance Sheet Liabilities: (1) Take or pay and throughput contracts, (2) Sales of receivables, (3) Securitizations, and (4) Joint ventures, subsidiary & affiliate contracts.

Sales-type Lease: Created by asset manufacturer and recognizes profit over life of contract, plus interest income, under operating lease. Profit recognized immediately under capital lease.

Direct Financing Lease: Created by a third party that buys the asset and leases it. Only interest income recognized.

SHAREHOLDERS' EQUITY

Contributed Capital = Par value of stock + Paid-in-capital excess of par. A stock may not have a par value, in which case it may have a stated value for legal reasons.

INCOME STATEMENT

Reporting Order: (1) Income from continuing operations, (2) Discontinued items, (3) Extraordinary items, and (4) Effect of accounting changes.

Stock Dividends & Splits: Have no economic value. Splits require a memorandum entry,

dividends require adjustment to contributed capital and retained earnings accounts.

STATEMENT OF CASH FLOWS

Basic Format: (1) CFO, (2) CFI, and (3) CFF. Net change in cash = CFO + CFI + CFF (signs respected). Ending cash = Beginning cash + Net change in cash.

Dividends & Interest Paid: Dividends are a part of CFF. Interest is a part of CFO. It is reclassified as CFF under functional method.

Gain & Loss on Sale of Assets: Part of CFI.

Goal of Financial Statements: Profitability not liquidity for investors and lenders.

Direct and Indirect Method: Direct method starts at the top, Revenues. Indirect methods starts at the bottom, Net Income. Reconciliation of cash flow and balance sheet affected by mergers and acquisitions.
Free Cash Flow = CFO – Capital expenditures required to maintain assets

ACCRUAL CONCEPT: INCOME & ASSETS

Principles & Goals:

- Matching principle for revenues and costs
- Analyst to focus on income from continuing operations

Revenue Recognition: (1) Time of sale, (2) Proportional recognition for advance receipts, (3) Percentage of completion (POC), (4) Completed contract (CC), (5) Installment sales, and (6) Cost recovery.

POC vs. CC: (1) Liabilities lower under POC, (2) Assets greater under POC, (3) Equity greater under POC, (4) Financial leverage lower under POC.

Managerial Discretion and Accounting Effects: (1) Good and bad news, (2) Big-bath accounting, (3) Income smoothing, (4) Accounting changes. These are related to discontinued operations, extraordinary items, and accounting changes.

Balance Sheet Assets: Recognized at lower of cost or market (conservatism).

FINANCIAL RATIOS & EPS

Common Size Income Statement & Balance Sheet: All income statement items divided by Revenues. All balance sheet items divided by Total Assets. Adjusts for size between firms and over time.

Significant Financial Ratios:

- *Current ratio* = Current Assets ÷ Current liabilities (CL)
- *Quick ratio* = [Cash + Marketable securities + Receivables] ÷ CL
- *Receivables days* = (Receivables ÷ Sales) *x* 365
- *Inventory turnover* = COGS÷Inventory
- *Inventory processing days* = 365 ÷ Inventory turnover
- *Inventory days* = (Inventory ÷ COGS) *x* 365
- *Payables payment period* = (Payables ÷ COGS) *x* 365
- *Cash conversion cycle* = Receivables days + Inventory processing days – Payables payment period.
- *Total Asset Turnover* = Sales ÷ Total assets
- *Gross margin* = Gross profit ÷ Sales
- *Net profit margin* = Net income (NI) ÷ Sales
- *Return on total capital employed* = [Net profit + Interest] ÷ Total capital
- *Return on equity* = Net income ÷ Equity
- *Operating leverage* = % change in operating profit ÷ % change in sales
- *Debt-equity ratio* = Long term debt ÷ Equity
- *Total debt ratio* = [Long term debt + CL] ÷ [Long term debt + CL + equity]
- *Interest coverage ratio* = [EBIT + Lease interest] ÷ Debt and lease interest
- *Cash flow coverage ratio* = [CFO + Debt interest and lease interest] ÷ debt interest and lease interest
- *Sustainable growth* = Retention ratio *x* ROE
- *DuPont analysis*: ROE = (NI/Sales) *x* (Sales/Total Assets) *x* (Total Assets/Equity)

Problems with Ratio Analysis: accounting differences, non-homogeneous firms, out-of-range values, inconsistency among ratios.

DILUTIVE SECURITIES & EPS

Simple Capital Structure: Only has common and preferred equity.

Complex Capital Structure: Has equity and convertible bonds, options, warrants etc.

Basic EPS = [Net income – Preferred dividends] ÷ Weighted average number of shares without adjusting for options etc.

Adjusting Weighted Number of Shares: (1) Total shares approach – all shares outstanding adjusted for period over which outstanding, (2) Incremental shares approach – additional shares adjusted for

period over which outstanding.

Adjustment for Stock Dividends and Splits: Applied as of beginning of fiscal year, i.e., previous share actions, not to shares issued, exercised or purchased after the split or dividend.

Convertible Bonds: Diluted EPS – include after-tax interest savings in numerator.

CORPORATE FINANCE

COST OF CAPITAL (COC)

After-tax Cost of Debt = k_d *x* (1-T)

Cost of Preferred Stock = $k_{ps} = D_{ps} \div P_{net}$

Cost of Retained Earnings:

- *CAPM/SML Approach*
  $k_s = k_{riskfree} + \beta_s [k_m - k_{riskfree}]$
- *Bond yield plus risk premium Approach*
  k_s = Yield on company's LT debt + Risk premium (between 3 and 5%)
- *DCF Approach*

$$k_s = \frac{D_1}{P_0} + g$$

Cost of New Equity:

$$k_{ne} = \frac{D_1}{P_0 (1 - F)} + g$$

Wtd. Average Cost of Capital (WACC):

$$k_a = w_d \, k_d \, (1-T) + w_{ps} \, k_{ps} + w_{c.\,equity} \, k_{c.\,equity}$$

Weights are based on market values.

Marginal Cost of Capital: It is computed with the help of WACC by plugging in marginal cost of each component of capital.

CAPITAL BUDGETING

Payback Period: Period over which the initial investment is recouped. Discounted payback discounts the cash flows for estimating the recouping period.

NPV Rule: Discount and add all cash flows (future and initial investment). Accept if NPV ≥ 0.

IRR Rule: Use the NPV equation and find the discount rate that sets NPV = 0. Accept if IRR ≥ Discount rate (cost of capital).

Conflicts Between IRR and NPV: Differences in timing and scale of mutually exclusive projects. Always go with NPV.

Multiple IRR Problem: It occurs when cash flows change signs more than once. Be careful for the range of COC matters.

NPV Profile: Graphical relationship between project NPV and different discount rates. Its intersection with the x-axis gives the IRR.

Incremental Cash Flow: Always use incremental cash flow for project analysis.

NWC = Current Assets – Current Liabilities.

Operating CF for a Project = Net income + Depreciation + Change in working capital

Terminal Cash Flow from Salvage = Salvage value – Tax rate *x* (Salvage value – Book value). Recovery of NWC is usually not taxed.

Unequal Lives: (1) Replacement chain – use the lowest common multiple of project lives and estimate their NPVs over this period. (2) Equivalent Annuity Approach –after computing NPV of a project find the equivalent annuity payment over project's life that has the same present value as NPV.

Effect of Inflation: Use nominal cash flows and discount them at nominal rates.

Types of Project Risk: (1) Standalone risk, (2) Corporate risk, and (3) Market risk.

Sensitivity Analysis: Changes in project NPV to changes in single inputs or variables.

Scenario Analysis: Project NPV under different scenarios where many variables change simultaneously.
Monte-Carlo Simulation: Extension of scenario analysis where thousands of scenarios are randomly generated with the help of computers.

CAPITAL STRUCTURE

Optimal Capital Structure: Mix of debt and equity that maximizes value of the firm.

Irrelevance Proposition: In the absence of taxes and bankruptcy costs, financial leverage is irrelevant to a firm.

Tradeoff Theory: Under bankruptcy and taxes, deductibility of interest shields taxes but higher debt levels increase cost of debt due to potential of bankruptcy. At the optimal leverage, the marginal tax benefits equal marginal bankruptcy cost.

Degree of Operating Leverage (DOL) = q(p – v) / [q(p – v) – F]. *q* is quantity sold, *p* is unit price, *v* is unit variable cost and *F* is total fixed cost. Equals percent change in EBIT for a percent change in sales.

Degree of Financial Leverage (DFL) = EBIT ÷ [EBIT – Interest]. Equals percent change in profit before tax for a percent change in EBIT.

Degree of Total Leverage (DTL) = DOL *x* DFL.

DTL = DTL = q(p – v) / [q(p – v) – F - I]

DIVIDEND POLICY

Dividend Irrelevance Theory: If a company has invested optimally it cannot increase its stock price by increasing dividends. The latter will hurt the stock price as it will have to curt back on investment.

Bird-in-hand theory: Investors prefer a dollar of dividends over a dollar of capital gains since the latter is uncertain. A company can increase its stock price by raising dividends.

Tax preference theory: Investors in high tax brackets prefer capital gains which are taxed at a lower rate. Investors in low tax brackets prefer higher dividends. A company can decide on a dividend policy depending upon which clientele it is targeting.

Estimating growth rate: Under constant growth and dividend payout, dividend growth rate = (1 – payout ratio) *x* ROE.

Signaling earnings: It claims that firms signal their future cash flows and earnings by changing their dividends. A brighter future is signaled by an increase in dividends and vice-versa.

Clientele Effect: Different types of investors gravitate to different firms – for capital gains or dividends, based on their dividend policies.

ASSET VALUATION

SECURITIES MARKETS

Call Market: Trading takes place at specific times and uses the auction process to match trades.

Continuous Market: Trading takes place at all times, and primarily uses dealers.

Combination Market: Has features of call market and continuous trading. Uses both auction process and dealer market.

Role of Specialist: On the NYSE, a specialist is responsible for making an orderly and continuous market in a security and is required to stand ready to buy on his or her account as a dealer. Acts both as a broker and a dealer. A broker only matches trades. A dealer buys on his or her account.

Types of Orders:

- Market order (executed at current price)
- Limit buy order (maximum purchase price specified below the current price)
- Limit sell order (minimum selling price specified above the current price)
- Stop loss sell order (a price specified below the current price to liquidate a long position)
- Stop loss buy (a price specified above the current price to reverse a current short position)

Price at Margin Call:

$$P_m = \frac{Loan\ amount}{No.of\ shares\ x\ (1 - MM)};$$

MM is maintenance margin.

Rate of Return on Margin Transaction =

$$\left(\frac{Sale\ price - Purchase\ price - Rate\ x\ Loan\ x\ Days}{Initial\ margin} \right) x \frac{360}{No.of\ days}; Total\ shares, or\ per\ share\ basis$$

SECURITY MARKET INDICATORS

Price-weighted Average (PWA):

$$PWA = [\Sigma_i P_i] / n$$

If a stock splits, the denominator, *n*, has to be adjusted. Over time it loses its meaning.

Value-weighted Index (VWI):

$I_t = [\Sigma MV_{i,t} / \Sigma MV_{i,t-1}] \ x \ I_{t-1}$; where $MV_{i,t}$ is market value of security *i* in period *t*.

Equally or Un-Weighted Index (EWI):

$I_t = [1 + (\Sigma_i \Delta_{i,t}) / n] \ x \ I_{t-1}$; where $\Delta_{i,t}$ is fractional change in value of security i.

Geometrically Weighted Index (GWI):

$I_t = [\Pi_i(1 + \Delta_{i,t})]^{1/n} \ x \ I_{t-1}$; where Π denotes product. $\Delta_{i,t}$ is same as before.

Biases: PWIs is biased toward high priced stocks; VWI toward high market value stocks; and, EWI toward smaller stocks.

MARKET EFFICIENCY

Efficient capital market: A market is informationally efficient if it instantaneously reflects all information in security prices.

Weak-form: Prices reflect all past information: past prices, volume, returns, volatility, and all transactions (specialists). Supported by tests autocorrelation and non-parametric runs tests.

Semi-strong form: Prices reflect past information and all currently available public information. Evidence casts doubt. Anomalies: calendar based, financial ratios, size based, and analyst following.

Strong form: Markets are semi-strong form efficient, and security prices also reflect all privately held information. Evidence mixed.

SECURITY VALUATION

Top Down Approach: Global Economic Analysis ⇒ Industry Analysis ⇒ Company Analysis.

Stock Valuation: R_t is future return.

$$V_0 = \sum_t \frac{R_t}{(1+r)^t}$$; r is discount rate.

Multi-period Dividend Discount Model:

$$V_0 = \frac{D_1}{(1+r_s)} + \cdots \frac{D_n}{(1+r_s)^n} + \frac{P_n}{(1+r_s)^n}$$

Constant Growth DDM:

$$V_0 = \frac{D_1}{(r_s - g)}$$

$$\frac{P_0}{E_1} = \frac{D_1 / E_1}{(r_s - g)} = \frac{PO}{(r_s - g)}$$

Stock with Supernormal Growth:

$$V_0 = \left(\sum_{i=1}^{T} \frac{D_i}{(1+r_s)^i} \right) + \frac{1}{(1+r_s)^T} \; x \; V_T$$, where

$$V_T = \frac{D_{T+1}}{r_s - g}$$

Discount Rate: *Required rate of return* = Real risk free rate + Inflation premium + Risk premium

Growth Rate of Earnings = RR *x* ROE; where RR is retention rate = (1 – payout).

STOCK MARKET SERIES

EPS of Market Series:

- *Estimate sales per share* = Last period sales + Projected change
- *Estimate operating margin*
  EBITDA per share = Operating margin *x* Sales per share
- *Estimate depreciation per share* = Depreciation rate *x* Estimated Property Plant and Equipment per share ⇒
  EBIT per share = EBITDA per share – Depreciation per share
- *Estimated interest per share* = Outstanding debt per share *x* Interest rate; Profit before taxes per share = EBIT per share – Interest per share
- Estimated EPS = Profit before taxes per share *x* (1– *T*); where *T* is the estimated average tax rate per share.

Expected Return on Stock Market Series:

$$\frac{End\ of\ period\ index - Current\ index + Exp\ dividends}{Current\ value}$$

INDUSTRY ANALYSIS

N-firm Concentration Ratio = $\sum_{i=1}^{N} ms_i$

Herfindahl Index:

$$H^{-1} = [\sum_{i=1}^{N} ms_i^2]^{-1}$$

COMPANY ANALYSIS

EPS of a Company: NPM = NI/Sales

$$EPS = \frac{NPM\ x\ S}{No.\ of\ Shares\ outstanding}$$

P-E ratio:

$$\frac{Payout\ Ratio\ (PO)}{Required\ rate\ of\ return\ (k_S) - growth\ (g)}$$

$$k_S = Risk\ free\ rate + \beta_S\ x\ Market\ risk\ premium$$

or

$$k_S = \frac{D_1}{P_0} + g$$

Value of a Company: $V_t = P\text{-}E_t$ x EPS_{t+1}

TECHNICAL ANALYSIS

Technical Analysts and Market Efficiency: Believe that (1) market values determined by supply and demand factors, and (2) supply and demand are influenced by rational and irrational factors.

Technicians Believe: (1) Existence of trends, and (2) Prices adjust gradually.

Other Differences: (1) data sources – technicians use market data, fundamentalists use non-market data, (2) fundamentalists concerned with security under and over-pricing and timing is critical, technicians believe trends take time to develop so there is no need to hurry, (3) technicians believe trends predict fundamental changes, and fundamentalists believe analysis predicts trends, (4) technicians believe markets are not (weak-form) efficient, fundamentalists believe markets are (weak-form) efficient.

Technicians' Tools: (1) smart money indicators, (2) contrary opinions, (market breadth indicators, and (4) price-volume series.

PRICE MULTIPLES

Different Measures:

- *P-E ratio*. Should be forward looking and adjusted for: business cycle phases, accounting methods, and dilutive securities.
- *Price-to-book*. Affected by business model (asset heavy) and inflation. Adjust for: goodwill and intangibles, off-balance sheet assets and liabilities, accounting methods (LIFO vs. FIFO), dilution effects.
- *Price-to-sales*. Affected by revenue recognition methods and business cycle.
- *Price-to-cash flow*. Cash flow per share = EPS + [Depreciation + Amortization + Depletion. Affected by revenue recognition methods, accounts receivables and bill-and-hold practices. Adjust for business cycle effects and impact of dilution.

DEBT INVESTMENTS

BOND BASICS

Features of a Bond:

- Principal or face (par) value
- Term to maturity
- Regular coupon payments (annual or semi-annual)
- Some bonds are callable at a fixed price above par, that declines with time
- Some bonds are putable where the investor can sell them to issuer before maturity
- Some bonds are convertible into common shares at pre-determined conversion price.

Special Redemption Price: Usually par value as in the case of sinking fund bonds and confiscation of corporate property by government.

Structure of Floating Rate Securities:

- *Quoted margin* above the reference rate
- *Deleveraged floater*, where the reference rate is multiplied by a factor less than before the margin is added
- *Cap rate*, where the maximum borrowing rate is fixed
- *Floor*, where the minimum lending rate is fixed
- *Collar*, where there is a cap and a floor.

Types of Floating Rate Securities:

- *Inverse floater*, where the coupon rate moves in a direction opposite to that of the reference rate
- *Dual indexed floater*, where the reference rate is the difference between two reference rates
- *Ratchet bond*, where the coupon can only adjust downward, not upward
- Stepped up floater where the quoted margin adjusts upward or downward over time.
- *Non-interest rate index floater*, where the reference rate is other than an interest rate, such as a commodity price, equity index, currency index etc. Used for hedging.

Accrued Interest: Bond price on a coupon payment date is called *clean price*. Interest accrues between bond payment dates.

Full Price = Clean price + Accrued interest.

Repo Agreement: A simultaneous sale and agreement to purchase a security by a dealer within a short period, to finance purchases.

BOND RISKS

Types of Risks:

- *Interest rate risk* – price fluctuations due to interest rate changes. Interest rates and bond prices move inversely
- *Call risk* – a bond will be called away below its intrinsic value
- *Prepayment risk* – a bond's principal will be paid early when the interest rates are lower
- *Yield curve risk* – that the yield curve will move in parallel or twist
- *Reinvestment risk* – bond coupon and

- amortization payments will be reinvested at lower rates
- *Credit risk* – that a bond may default on its promise to make coupon and principal payments
- *Liquidity risk* – that a bond cannot be sold at fair value quickly. Measured by bid ask spread = Ask – Bid.
- *Exchange rate risk* – that the foreign currency in which bond cash flows are paid will get weaker relative to domestic currency
- *Volatility risk* – higher interest rate volatility will raise or lower the price of an embedded option, adversely affecting the bond price
- *Inflation risk* – that higher inflation would reduce the purchasing power of future cash flows
- *Event risk* – that an issuer specific event will affect the bond price.

Price Relative to Par: (1) *Par* bond – where Price = Par (coupon = market rate); (2) *Premium* bond – where Price > Par (coupon > market rate); and (3) *Discount* bond – where Price < Par (coupon < market rate).

Features & Interest Rate Sensitivity:

- Longer bonds more are sensitive
- Higher coupon bonds are less sensitive
- Callable bonds are less sensitive than non-callable bonds.

Option Embedded Bonds:

Callable bond price = Price of equivalent straight bond – Call option value

Putable bond price = Non-putable bond price + Put option price

Bond Duration: Price sensitivity to interest rates. Measures percent change in bond price for a one percent change in interest rates.

Bond duration =

$$\frac{\text{Price with yield decrease} - \text{Price with yield increase}}{2 \times \text{Original Price} \times \text{Basis point change in yield}}$$

Calculating Bond Price Change:

$$\text{Estimated \% change} = \frac{Duration}{100} \times bp\ change$$

Price change is always opposite to the change in interest rate.

BOND SECTORS

International bonds:

- *Foreign bonds* – issued by a foreign entity from investor's perspective
- *Eurobonds* – unregistered bonds issued outside the jurisdiction of any national government; offered to investors in several countries simultaneously
- *Global bonds* – issued by foreign borrowers in the currency's home country as well as in eurobond market
- *Sovereign debt* are borrowings of national governments

On-the-run securities: Most recently auctioned US Treasury securities and have the highest liquidity.

Stripped securities: Coupon and principal payments stripped from the original bond according to the maturity date.

YIELD CURVE

Term Structure Theories: (1) Pure expectations – Forward rates are expected future one-period spot rates. (2) Liquidity preference – Longer maturity bonds are more sensitive to interest rate changes and this demand higher premium known as liquidity premium. It is a rising curve. (3) Segmentation hypothesis – Bond maturities are divided into separate segments with their own supply-demand curves. Preferred habitat argues that participants may move close to their segment to produce a smooth yield curve that removes breaks or arbitrage opportunities.

Absolute yield spread = Yield on a Bond – Yield on Benchmark Bond

Relative Yield Spread

$$= \frac{\text{Yield on Bond} - \text{Yield on Benchmark Bond}}{\text{Yield on Benchmark Bond}}$$

Yield Ratio =

$$\frac{\text{Yield on Bond}}{\text{Yield on Benchmark Bond}} = 1 + \text{Relative yield spread}$$

Yields on Munis: *Tax-equivalent yield* = Tax-exempt yield ÷ (1 – marginal tax rate)

BOND VALUATION

Arbitrage Free Valuation: Value a bond using spot yield curve, with its coupon and principal payments as stripped securities. Existence of "Treasury strip" security market is the *key* to reaching equilibrium

If Sum of strip values > Bond price ⇒ Buy the bond & strip coupons, and sell separately

If Sum of strip values < Bond price ⇒ Buy the strips & sell as a reconstitute bond.

Types of Yields:

- Current Yield = $\frac{\text{annual coupon interest}}{\text{current price}}$
- Yield to maturity
- Yield to first call
- Yield to first par call date
- Yield to refunding
- Yield to put
- Yield to worst
- Cash flow yield (for MBS)

Convert Semi-annual Yield to Ann. Pay:

$$\left(1+\frac{bond-equivalent\ yield}{2}\right)^2-1$$

Convert Ann. Yield to Semi-annual pay:

$$2x[(1+annual-pay\ yield)^{0.5}-1]$$

Value of a Bond Using Spot Rates:

$$Price=\frac{C/2}{\left(1+\frac{y_1}{2}\right)^1}+\frac{C/2}{\left(1+\frac{y_2}{2}\right)^2}+\ldots+\frac{C/2}{\left(1+\frac{y_T}{2}\right)^T}+\frac{F}{\left(1+\frac{y_T}{2}\right)^T}$$

y_t = Treasury spot rate for period t + spread

Bootstrapping Method for Yield Curve:

$$P_0^1=\frac{C_1+FV}{(1+YTM_1)};\ P_0^2=\frac{C_2}{(1+YTM_1)}+\frac{C_2+FV}{(1+S_2)^2}$$

Option Cost = z-spread – OAS

Bond Value Using Forward rates:

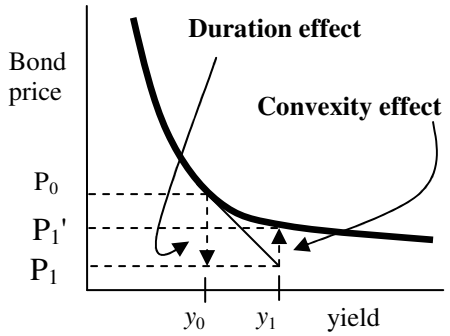
$$\frac{C_T}{(1+f_1)(1+f_2)(1+f_3)\ldots(1+f_{T-1})(1+f_T)}$$

Compute Forward Rate Given Spot Rate:

$$f_t=\frac{(1+z_t)^t}{(1+z_1)(1+f_2)\ldots(1+f_{t-2})(1+f_{t-1})}-1$$

$$=\frac{(1+z_t)^t}{(1+z_{t-1})^{t-1}}$$

Positive Convexity:



Total bond price change = – **Duration** effect + **Convexity** effect

Effective Duration = $\frac{P_- - P_+}{2xP_0x\Delta y}$

Percentage change in price ≈ Effective duration x Δy x 100

Modified Duration = $-\frac{dP}{P}x\frac{1}{dy}=$

$$\frac{\text{Macaulay duration}}{(1+y/n)}$$

Portfolio Duration = $\Sigma w_i D_i$

Convexity Adjustment: = + C x $(\Delta y)^2$

Net % change = [– D x Δy + C x $(\Delta y)^2$] x 100

Price Value of Basis Point (PVBP):

dP = D x 0.0001 x P

DERIVATIVE INVESTMENTS

Forward Contract and Arbitrage:

Forward price = Current price + Risk free interest = P_0 x $(1 + R_F)$

FRA: It is a forward contract on an interest rate, such as LIBOR.

Payment on an FRA:

$$\frac{(\textit{Expiration day rate} - \textit{FRA rate})\ x\ \textit{Notional principal}\ x\ \dfrac{\textit{No. of days in underlying rate}}{360}}{\left(1 + \dfrac{\textit{FRA rate}}{100}\ x\ \dfrac{\textit{No. of days in underlying rate}}{360}\right)}$$

Variation margin: Shortfall amount that needs to be deposited by the next trading day to bring the margin *back up to initial margin* for a futures contract, in the event of a margin call is called variation margin.

Futures Delivery Option: The short has the right to determine –

- *what* to deliver,
- *when* to deliver, and
- *where* to deliver

OPTION CONTRACTS

Intrinsic Value:

Calls: Payoff = Maximum (0, Stock price – Exercise price)

Puts: Maximum (0, Exercise price – Stock price)

Time Value of Option = Option premium – Intrinsic value

Zero-sum Game: *Total payoff under an option* = Payoff to LONG + Payoff to SHORT = 0

Interest Rate Options:

Call: Payoff = Maximum(0, Interest rate – Strike rate) x Notional Principal $x \dfrac{\text{Days in rate}}{360}$

Put: Payoff = Maximum(0, Strike rate – Interest rate) x Notional Principal $x \dfrac{\text{Days in rate}}{360}$

FRA and Options: A FRA is equivalent to a long call on the LIBOR and a short put on the LIBOR with the same strike rate.

Interest Rate Caps and Floors: A Cap is a series of caplets and a Floor is a series of floorlets. A caplet is a call option on an interest rate, and a floorlet is a put option on an interest rate.

Lower Bounds & Maximum Value:

Option type	Modified lower bounds
European call	$\text{Max}(0, S_0 - X/(1+r_F)^T)$
European put	$\text{Max}(0, X/(1+r_F)^T - S_0)$
American call	$\text{Max}(0, S_0 - X/(1+r_F)^T)$
American put	$\text{Max}(0, X - S_0)$

Option type	Maximum value
European call	S_0
European put	$X/(1+r_F)^T$
American call	S_0
American put	X

Options and Exercise Price: A call with a lower exercise price is worth more. A put with a higher exercise price is worth more.

Time to Expiration: Longer time is beneficial for calls. It is also beneficial for American puts. Inconclusive for European puts due to waiting disadvantage.

Put-Call Parity:

$c_0 + X/(1+r_F)^T = p_0 + S_0$

Modified Put-Call Parity:

$c_0 + X/(1+r_F)^T = p_0 + [S_0 - PV_0(CF_t: t = 0, 1, 2, \ldots T)]$

Maximum Profit, Loss and Breakeven:

Option	Max Profit	Max Loss	B-Even Price
+Call	∞	$-c_0$	$X + c_0$
–Call	c_0	$-\infty$	$X + c_0$
+Put	$X - p_0$	$-p_0$	$X - p_0$
–Put	p_0	$-X + p_0$	$X - p_0$
Cov Call	c_0	$-X + c_0$	$X - c_0$
Pr Put	∞	$-p_0$	$X + p_0$

ALTERNATIVE INVESTMENTS

Mutual Funds: NAV = $\dfrac{\text{Mkt.Val.of Assets - Mkt.Val.of Liabilities}}{\text{No.of Shares Outstanding}}$

Real Estate Valuation:

- *Cost approach*
- *NOI Approach*

$$\text{Appraised value} = \frac{\text{NOI}}{\text{Market cap rate}}$$

$$\text{Maket cap rate} = \frac{\text{Benchmark property NOI}}{\text{Benchmark property price}}$$

- *Discounted cash flow approach*
- *Sales comparison (Hedonistic approach)*

Venture Capital Evaluation:

Expected NPV = Prob (success) x NPV (success) + Prob (failure) x NPV (failure)

PORTFOLIO MANAGEMENT

CAPITAL MARKET THEORY

Required rate of return for security S:

$RRR_S = RRFRR + IP + RP_S$

IP = inflation premium, RP_S is risk premium. RRFRR is the real risk free rate of return. Under certainty, RRFRR is related to economic growth.

Nominal Risk Free Rate: NRFRR = (1+ RRFRR) *x* (1 + IP) – 1

Fundamental Risk Factors: (1) Business risk, (2) Financial risk, (3) Liquidity risk, (4) Exchange rate risk, and (5) Country risk.

Security Market Line (SML):

$k_S = R_F + \beta_S (k_M - R_F)$, where β_S is beta or market risk, k_M is the required rate of return on the market portfolio and R_F is the risk free rate of return.

Changes in Slope: The slope of SML changes when the risk premium ($k_M - R_F$) demanded by the market changes.

Shifts in SML: The SML shifts or its *y*-axis intercept changes when the risk free rate, R_F, changes, either due to the change in the rate of inflation (more common) or change in the real risk free rate of return.

Movements Along the SML: A security moves up or down along the SML based on its market risk, β_S. Beta of the market is 1.

ASSET ALLOCATION & IPS

IPS: An individual's Individual Policy Statement IPS is the blueprint for management of his or portfolio. It contains: Risk and return objectives and five constraints: time horizon, tax situation, liquidity needs, regulatory issues, and unique circumstances.

Risk Tolerance Factors: Wealth, Age, Expected income, and, Family situation.

Return Objectives: (1) Capital preservation, (2) Current income, (3) Capital appreciation, and (4) Total return.

PORTFOLIO MANAGEMENT

Portfolio Efficiency: A portfolio that offers the highest return for a given risk level, or has the lowest risk level for a given return.

Expected Return on a Security:

$$ER_j = \sum_{S=1}^{n} p_S \, x \, R_S$$

Expected Return on Portfolio =

$$ER_P = \sum_{j=1}^{N} w_j \, x \, ER_j$$

Variance of Security:

$$V = \sum_{S=1}^{n} p_S \, x [R_S - ER]^2$$

Correlation Coefficient:

$$\text{Corrln coefficient} = \frac{Covariance}{SD_{Stock\,A} \, x \, SD_{Stock\,B}}$$

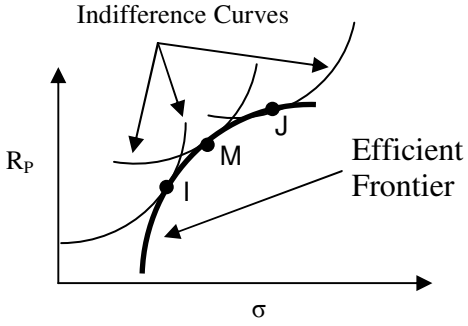
Covariance:

$$Cov_{AB} = \frac{\sum_{i=1}^{T} [X_{Ai} - EX_A] x [X_{Bi} - EX_B]}{T - 1}$$

Portfolio Variance:

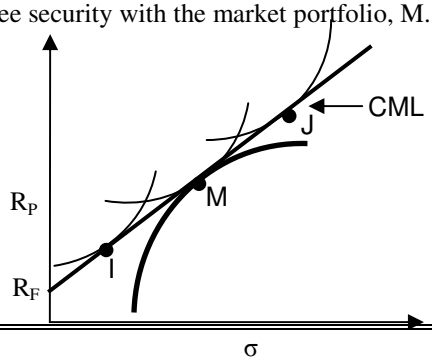
$$V_P = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B Cov_{AB}$$

Optimal Portfolio:



I and J are two different investors. M is the market portfolio. Each investor maximizes utility by finding a point on the highest indifference curve that is tangent to the efficient frontier.

Capital Market Line (CML): When a risk free security is introduced, a new efficient portfolio is created, known as CML. Investors move up to higher indifference curves by combining the risk free security with the market portfolio, M.



Expected Portfolio Return: With a risk free and risky security.

$$ER_P = w_F \, x \, R_F + (1 - w_F) \, x \, R_2$$

Portfolio Variance: $V_P = w_B^2 \sigma_B^2$, because the risk free security has zero variance.

Diversifiable and Non-diversifiable Risk:

Total risk = Systematic risk + Non-systematic (*diversifiable*) risk

Security Market Line:

$$R_S = R_F + \frac{Cov_{SM}}{\sigma^2_M}(R_M - R_F)$$

slope = $(R_M - R_F)$, and $\beta_S = \frac{Cov_{SM}}{\sigma^2_M}$

Characteristic Line: This is simply a regression between the return on a security and the market return based on historic data.

$$R_{St} = \alpha_S + \beta_S \, R_{Mt} + \varepsilon_{St}$$

The slope estimates the security's *beta*.

ETHICS & PROFESSIONAL STANDARDS

I. Professionalism.
A. Knowledge of the Law.
B. Independence and Objectivity.
C. Misrepresentation.
D. Misconduct.

II. Integrity of capital markets
A. Material Nonpublic Information.
B. Market Manipulation.

III. Duties to Clients.
A. Loyalty, Prudence, and Care.
B. Fair Dealing.
C. Suitability.
D. Performance Presentation.
E. Preservation of Confidentiality.

IV. Duties to Employers.
A. Loyalty.
B. Additional Compensation Arrangements.
C. Responsibilities of Supervisors.

V. Investment Analysis, Recommendations, and Actions.
A. Diligence and Reasonable Basis.
B. Communication with Clients and Prospective Clients. C. Record Retention.

VI. Conflicts of Interest.
A. Disclosure of Conflicts.
B. Priority of Transactions.
C. Referral fees.

VII. Responsibilities as a CFA Institute member or CFA Candidate.
A. Conduct as members and candidates in the CFA program.
B. Reference to CFA Institute, the CFA designation, and the CFA program.

Global Investment Performance Standards